

Notations used by TPS and ETPS

$[\sim \mathbf{A}]$ means “ \mathbf{A} is not true”.

$[\mathbf{A} \wedge \mathbf{B}]$ means “ \mathbf{A} and \mathbf{B} ”.

$[\mathbf{A} \vee \mathbf{B}]$ means “ \mathbf{A} or \mathbf{B} ”.

$[\mathbf{A} \supset \mathbf{B}]$ means “ \mathbf{A} implies \mathbf{B} ”.

$[\mathbf{A} \equiv \mathbf{B}]$ means “ \mathbf{A} if and only if \mathbf{B} ”.

$[\forall x \mathbf{A}]$ means “For every x , \mathbf{A} is true”.

$[\exists x \mathbf{A}]$ means “There exists an x such that \mathbf{A} is true”.

Bracket and Parenthesis Conventions

Outermost brackets and parentheses may be omitted.

Use the convention of association to the left. Thus,

A \supset **B** \supset **C** stands for $[[\mathbf{A} \supset \mathbf{B}] \supset \mathbf{C}]$.

A \supset **B** \supset **C** \supset **D** stands for $[[[\mathbf{A} \supset \mathbf{B}] \supset \mathbf{C}] \supset \mathbf{D}]$.

A dot stands for a left bracket, whose mate is as far to the right as is possible without altering the pairing of left and right brackets already present.

Thus,

A \supset **.****B** \supset **C** stands for [**A** \supset [**B** \supset **C**]].

[**A** \supset **.****B** \supset **C**] \supset **D** stands for [[**A** \supset [**B** \supset **C**]] \supset **D**].

When the relative scopes of several connectives of different kinds must be determined, \sim is to be given the smallest possible scope, then \wedge the next smallest possible scope except for \sim , then \vee , then \supset , then \equiv .

Church's Type Theory

Formulas of higher-order logic are written using the notation introduced in Alonzo Church's paper "A Formulation of the Simple Theory of Types", *Journal of Symbolic Logic* 5 (1940), 56-68.

$(\alpha\beta)$ is the type of functions
to objects of type α
from objects of type β .

This is sometimes written $(\beta \rightarrow \alpha)$.

Thus, if Y_α is the result of applying the function $F_{\alpha\beta}$ to the
argument X_β , we write

$$Y_\alpha = F_{\alpha\beta} X_\beta$$

By the convention of association to the left,
 $\alpha\beta\gamma$ stands for $((\alpha\beta)\gamma)$.

A function of two arguments can be represented as a function of one argument whose values are functions.

$$Z_\alpha = [[G_{((\alpha\beta)\gamma)}X_\gamma]Y_\beta] = G_{\alpha\beta\gamma}X_\gamma Y_\beta$$

An entity of type $((\alpha\beta)\gamma)$ may be regarded both as

a function mapping elements of type γ to functions of type $(\alpha\beta)$

and as

a function of two arguments (of types γ and β) which has values of type α .

o is the type of truth values and statements.

We identify a set of elements of type β with the function $S_{o\beta}$ which maps the elements in the set to truth and all other objects of type β to falsehood, and refer to $S_{o\beta}$ as a set. Thus:

$S_{o\beta} x_\beta$ means that $S_{o\beta} x_\beta$ is true.

$S_{o\beta} x_\beta$ means that $x_\beta \in S_{o\beta}$.

$S_{o\beta} = \{x_\beta \mid S_{o\beta} x_\beta\}$.

Similarly, $R_{o\beta\alpha}$ is a relation between objects of type α and objects of type β .

λ -Notation

$[\lambda v A(v)]$ denotes the function whose value for any argument v is $A(v)$.

For example, if $F(v) = v^2 + v + 5$ for all natural numbers v , then $F = [\lambda v . v^2 + v + 5]$

If $A(v)$ is a statement about v , $[\lambda v A(v)]$ denotes $\{v \mid A(v)\}$.

If $A(u, v)$ is a statement about u and v , $[\lambda u \lambda v A(u, v)]$ denotes $\{ \langle u, v \rangle \mid A(u, v) \}$.

λ -Conversion

If we apply the function denoted by $[\lambda v A(v)]$ to an argument denoted by W , the result may be denoted by $A(W)$.

For example, $[\lambda v . v^2 + v + 5] 7 = 7^2 + 7 + 5$

The replacement of $[\lambda v A(v)] W$ by $A(W)$ is called λ -conversion.

If $A(v)$ is a statement about v ,

$[\lambda v A(v)] W$ means

$W \in \{v | A(v)\}$, or $A(W)$.