Notations used by TPS and ETPS

[$\sim$ **A**] means "**A** is not true".

[**A** $\wedge$ **B**] means "**A** and **B**".

[**A** $\vee$ **B**] means "**A** or **B**".

[**A** $\supset$ **B**] means "**A** implies **B**".

[**A** $\equiv$ **B**] means "**A** if and only if **B**".

[$\forall$**x** **A**] means "For every **x**, **A** is true".

[$\exists$**x** **A**] means "There exists an **x** such that **A** is true".

Bracket and Parenthesis Conventions

Outermost brackets and parentheses may be omitted.

Use the convention of association to the left. Thus,

$\mathbf{A} \supset \mathbf{B} \supset \mathbf{C}$ stands for $[[\mathbf{A} \supset \mathbf{B}] \supset \mathbf{C}]$.
$\mathbf{A} \supset \mathbf{B} \supset \mathbf{C} \supset \mathbf{D}$ stands for $[[[\mathbf{A} \supset \mathbf{B}] \supset \mathbf{C}] \supset \mathbf{D}]$.

A dot stands for a left bracket, whose mate is as far to the right as is possible without altering the pairing of left and right brackets already present.

Thus,

$\mathbf{A} \supset .\mathbf{B} \supset \mathbf{C}$ stands for $[\mathbf{A} \supset [\mathbf{B} \supset \mathbf{C}]]$.

$[\mathbf{A} \supset .\mathbf{B} \supset \mathbf{C}] \supset \mathbf{D}$ stands for $[[\mathbf{A} \supset [\mathbf{B} \supset \mathbf{C}]] \supset \mathbf{D}]$.

When the relative scopes of several connectives of different kinds must be determined, $\sim$ is to be given the smallest possible scope, then $\wedge$ the next smallest possible scope except for $\sim$, then $\vee$, then $\supset$, then $\equiv$.

Church's Type Theory

Formulas of higher-order logic are written using the notation introduced in Alonzo Church's paper
"A Formulation of the Simple Theory of Types",
Journal of Symbolic Logic 5 (1940), 56-68.

$(\alpha\beta)$ is the type of functions
to objects of type α
from objects of type β.

This is sometimes written $(\beta \to \alpha)$.

Thus, if Y_α is the result of applying the function $F_{\alpha\beta}$ to the argument X_β, we write

$$Y_\alpha = F_{\alpha\beta}\, X_\beta$$

By the convention of association to the left,
$\alpha\beta\gamma$ stands for $((\alpha\beta)\gamma)$.

A function of two arguments can be represented as a function of one argument whose values are functions.

$$Z_\alpha = [[G_{((\alpha\beta)\gamma)} X_\gamma] Y_\beta] = G_{\alpha\beta\gamma} X_\gamma Y_\beta$$

An entity of type $((\alpha\beta)\gamma)$ may be regarded both as

a function mapping elements of type γ to functions of type $(\alpha\beta)$

and as

a function of two arguments (of types γ and β) which has values of type α.

o is the type of truth values and statements.

We identify a set of elements of type β with the function $S_{o\beta}$ which maps the elements in the set to truth and all other objects of type β to falsehood, and refer to $S_{o\beta}$ as a set. Thus:

$S_{o\beta}\, x_\beta$ means that $S_{o\beta}\, x_\beta$ is true.

$S_{o\beta}\, x_\beta$ means that $x_\beta \in S_{o\beta}$.

$S_{o\beta} = \{x_\beta \mid S_{o\beta}\, x_\beta\}$.

Similarly, $R_{o\beta\alpha}$ is a relation between objects of type α and objects of type β.

λ-Notation

$[\lambda v\ A(v)]$ denotes the function
whose value for any argument v is $A(v)$.

For example, if $F(v) = v^2 + v + 5$
for all natural numbers v,
then $F = [\lambda v\ .\ v^2 + v + 5]$

If $A(v)$ is a statement about v,
$[\lambda v\ A(v)]$ denotes $\{v \mid A(v)\}$.

If $A(u, v)$ is a statement about u and v,
$[\lambda u \lambda v\ A(u, v)]$ denotes $\{< u, v > \mid A(u, v)\}$.

λ-Conversion

If we apply the function denoted by $[\lambda v A(v)]$ to an argument denoted by W, the result may be denoted by $A(W)$.

For example, $[\lambda v \; . \; v^2 + v + 5]\, 7 = 7^2 + 7 + 5$

The replacement of $[\lambda v A(v)]\, W$ by $A(W)$ is called λ-conversion.

If $A(v)$ is a statement about v,
$[\lambda v A(v)]\, W$ means
$W \in \{v | A(v)\}$, or $A(W)$.